Analysis of the PeerRank Method for Peer Grading

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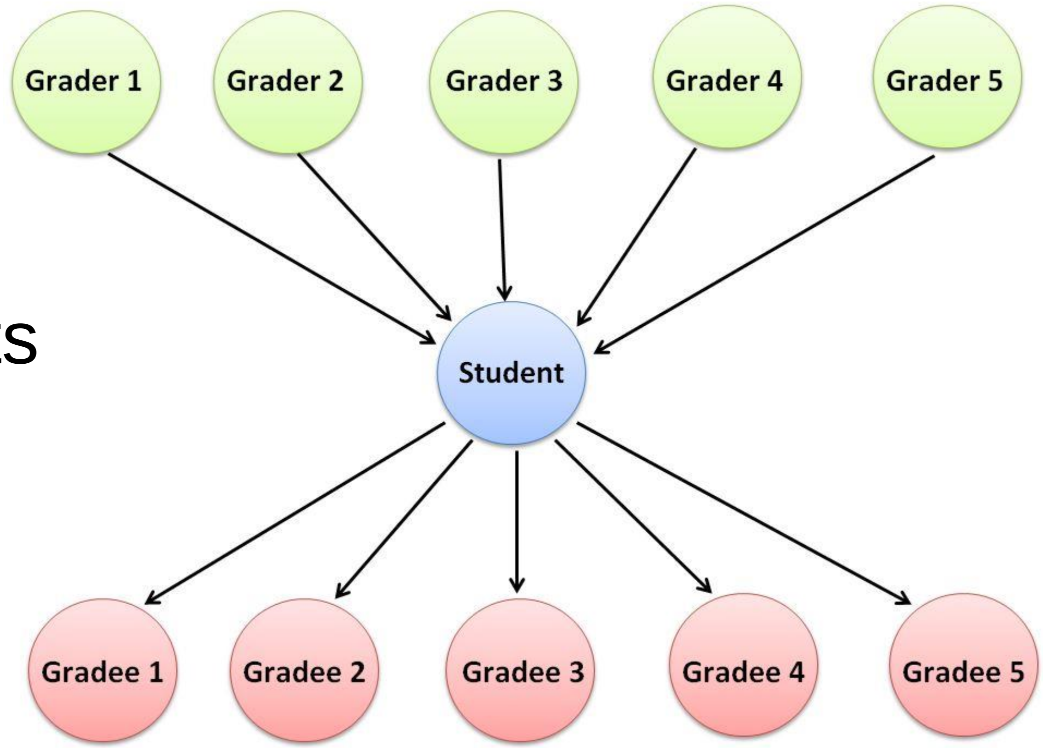
Benefits of Peer Grading

- Reduces time instructors spend grading
- Provides faster feedback for students
- Increases student understanding through analysis of others



Potential Issues with Peer Grading

<u>Issues:</u>

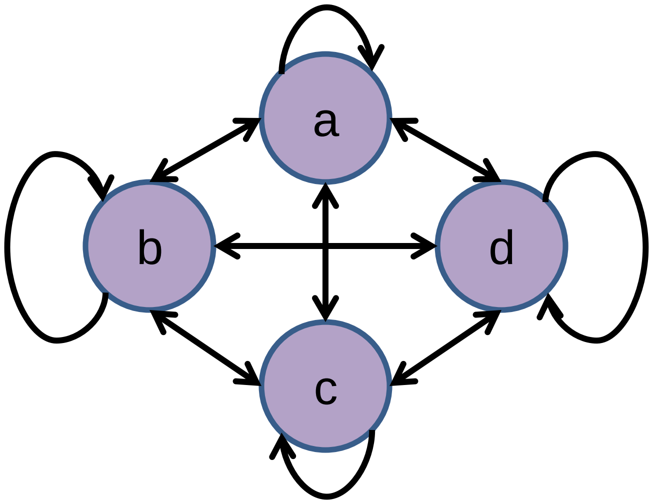
- Students may be unreliable graders
  - Inexperience in grading
  - Lack of understanding of material
- Students may not care about grading accurately

<u>Ways to Address:</u>

- Make inaccurate graders count less toward final grade
- Provide graders with an incentive to grade accurately

PeerRank

- Algorithm developed by Toby Walsh
- Two factors in final grade:
    - Weighted combination of grades from peers
    - Individual's own accuracy in grading others
- Same linear algebra foundations as Google PageRank
- Original application: Reviewing grant proposals



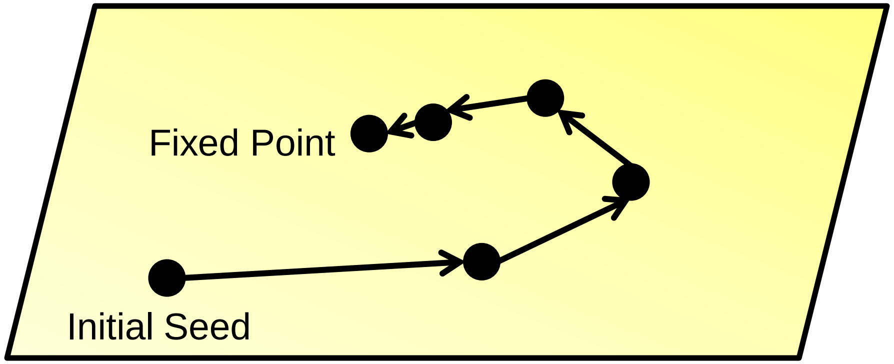
$$A = \begin{bmatrix} A_{a,a} & A_{a,b} & A_{a,c} & A_{a,d} \\ A_{b,a} & A_{b,b} & A_{b,c} & A_{b,d} \\ A_{c,a} & A_{c,b} & A_{c,c} & A_{c,d} \\ A_{d,a} & A_{d,b} & A_{d,c} & A_{d,d} \end{bmatrix}$$

PeerRank

$$X_i^0 = \frac{1}{m}\sum_j A_{i,j}$$

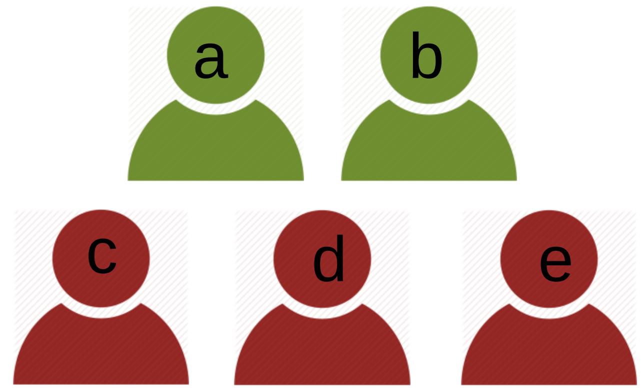
$$X_i^{n+1} = (1-\alpha-\beta)\cdot X_i^n + \frac{\alpha}{\sum_j X_j^n}\cdot \sum_j X_j^n \cdot A_{i,j} + \frac{\beta}{m}\cdot \sum_j 1 - \left|A_{j,i} - X_j^n\right|$$

- Start with "initial seed" grade vector $\overrightarrow{X^0}$
  - Average of grades received
- PeerRank equation is evaluated iteratively until fixed point is reached
  - $\overrightarrow{X^{n+1}} \approx \overrightarrow{X^n}$



Problems with PeerRank

- **Walsh's Assumption:** A grader's accuracy is assumed to be equal to their grade
  - Unrealistic assumption?
- No way of specifying "correctness"
  - May produce incorrect results



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Correct Result: $[1,1,0,0,0]$

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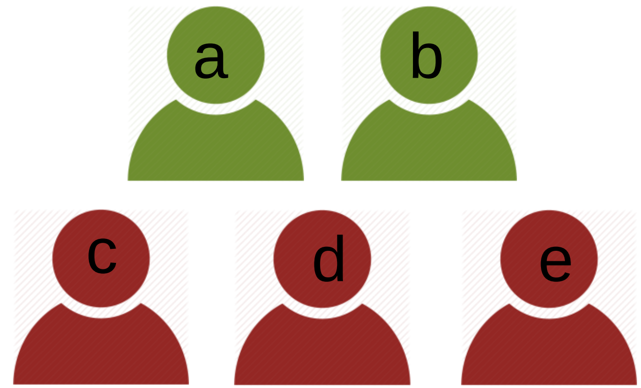

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Actual Result: [0,0,1,1,1]

Project Goal

Modify and adapt the PeerRank algorithm so that it can better provide accurate peer grading in a classroom setting

Incorporating “Ground Truth”

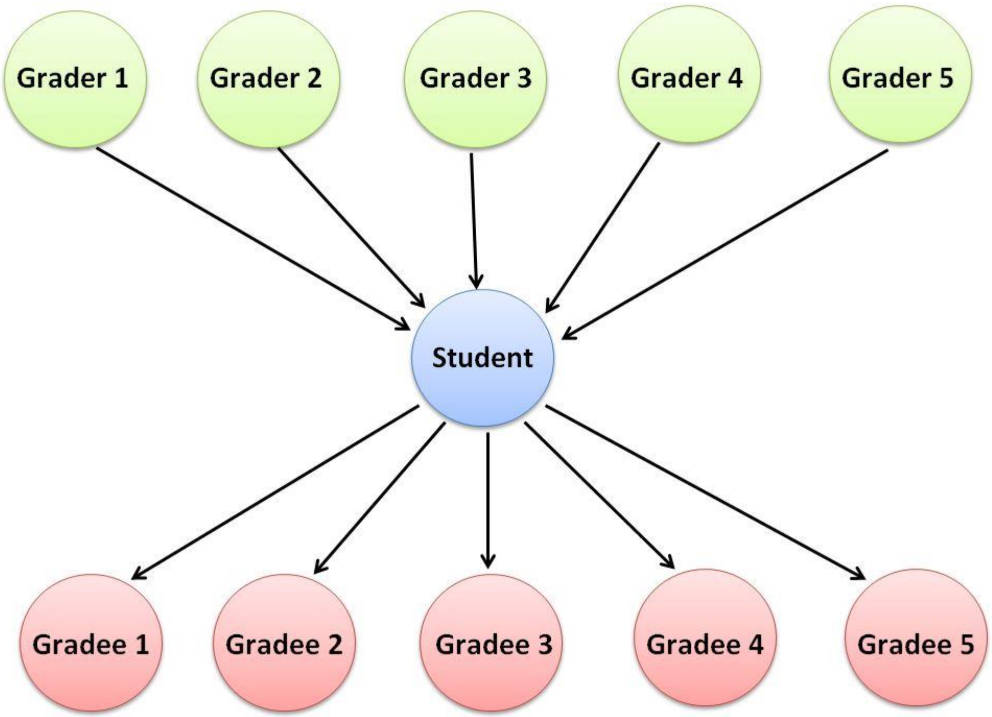
- Recall: There is no way of specifying “correctness” in PeerRank.

- In education, there is a notion of “ground truth” in assignments
  - Right answer vs. wrong answer
  - Correct proof
  - Essay with strong argument and no errors

- Ground truth is normally determined by instructor

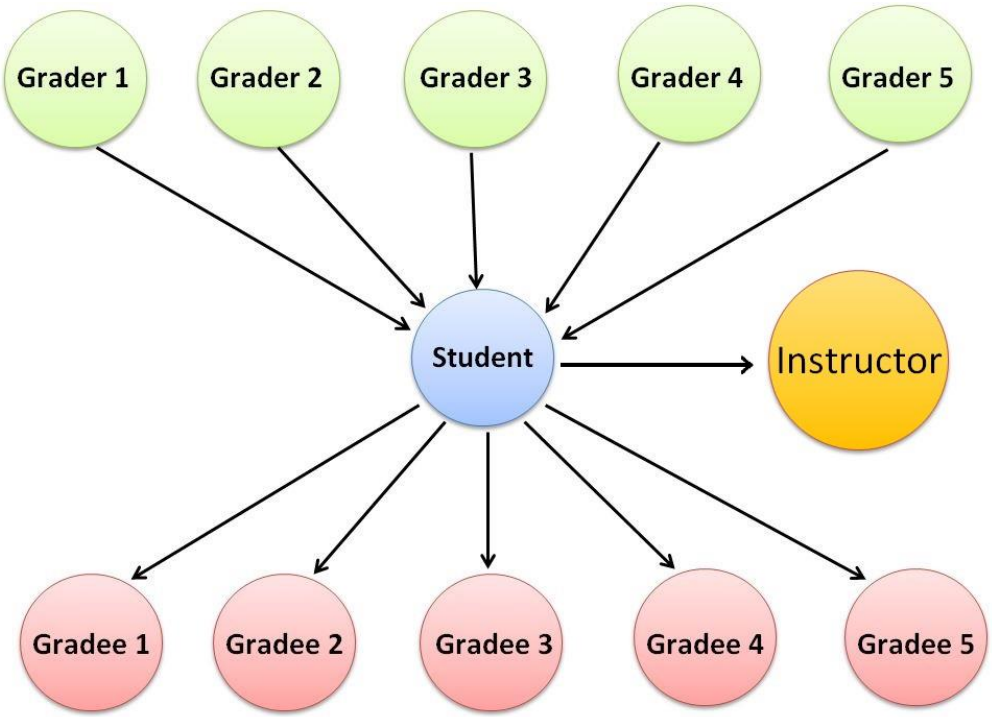
Incorporating "Ground Truth"

- **Goal:** Give the instructor a role in the PeerRank process that influences the accuracy weights of the students



Incorporating “Ground Truth”

- **Goal:** Give the instructor a role in the PeerRank process that influences the accuracy weights of the students
- **Solution:**
  - The instructor submits their own assignment for which they know the correct grade
  - Each student grades the instructor’s assignment, and their grading error determines their accuracy
    - Students do not know which assignment is instructor’s
  - Use these accuracies to produce a weighted combination of the peer grades



Our Method vs. PeerRank

<u>PeerRank:</u>

- Accuracy equal to grade
  - Walsh's assumption applies
- Iterative process
  - Final grades are fixed point

$$X_i^0 = \frac{1}{m}\sum_j A_{i,j}$$

$$X_i^{n+1} = (1-\alpha-\beta)\cdot X_i^n + \frac{\alpha}{\sum_j X_j^n}\cdot \sum_j X_j^n \cdot A_{i,j} + \frac{\beta}{m}\cdot \sum_j 1 - \left|A_{j,i} - X_j^n\right|$$

<u>Our Method:</u>

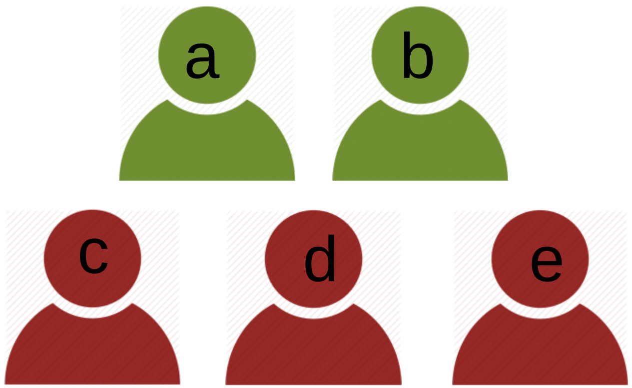
- Accuracy determined by accuracy in grading the instructor
  - Walsh's assumption no longer applies
- Non-iterative
  - Final grades are a weighted average of the peer grades, weighted by the accuracies

$$ACC_i = 1 - |A_{I,i} - X_I|$$

$$\vec{X} = \frac{1}{\left\|\overrightarrow{ACC}\right\|_1}\left(A \cdot \overrightarrow{ACC}\right)$$

Majority vs. Minority Case

- Recall: If a group of incorrect students outnumber a group of correct students, the wrong grades are produced by PeerRank.



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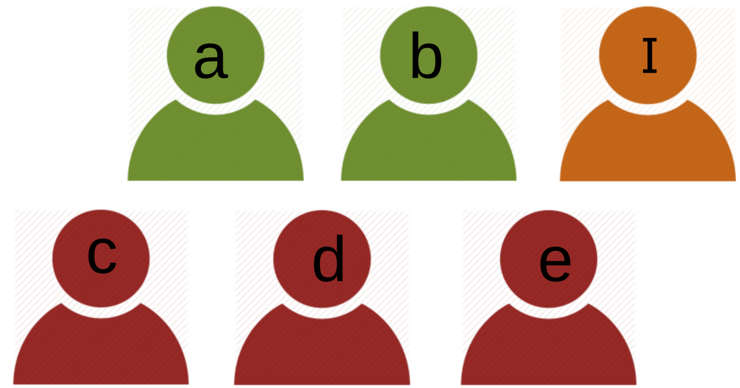

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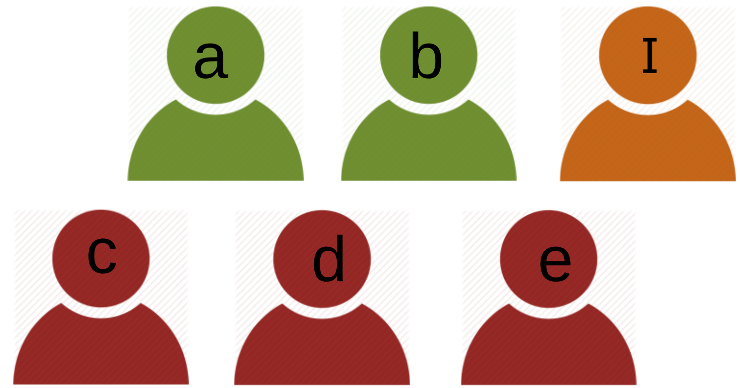

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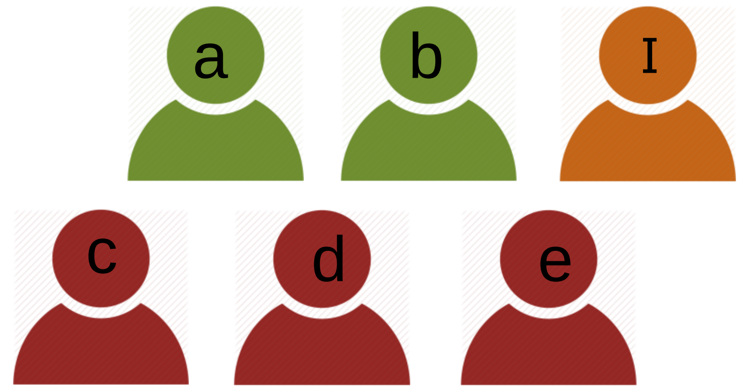

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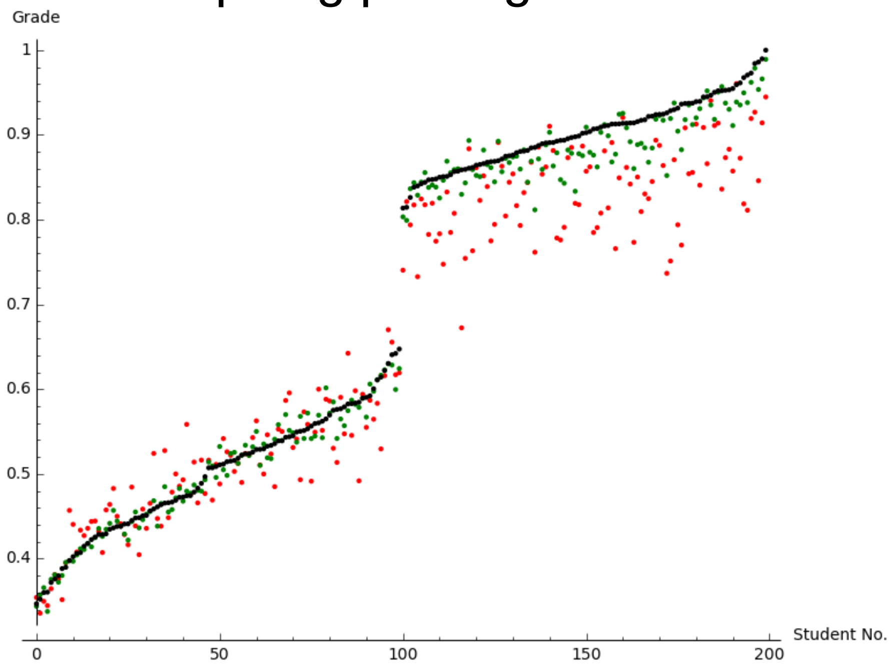
Correct Result: [1,1,0,0,0,1]

Accuracies: [1,1,0,0,0,1]

Actual Result: [1,1,0,0,0,1]

Implementation

- Algorithms for PeerRank and our method implemented in Sage
  - Based on Python
  - Additional math operations, including matrices and vectors
  - Graphing packages



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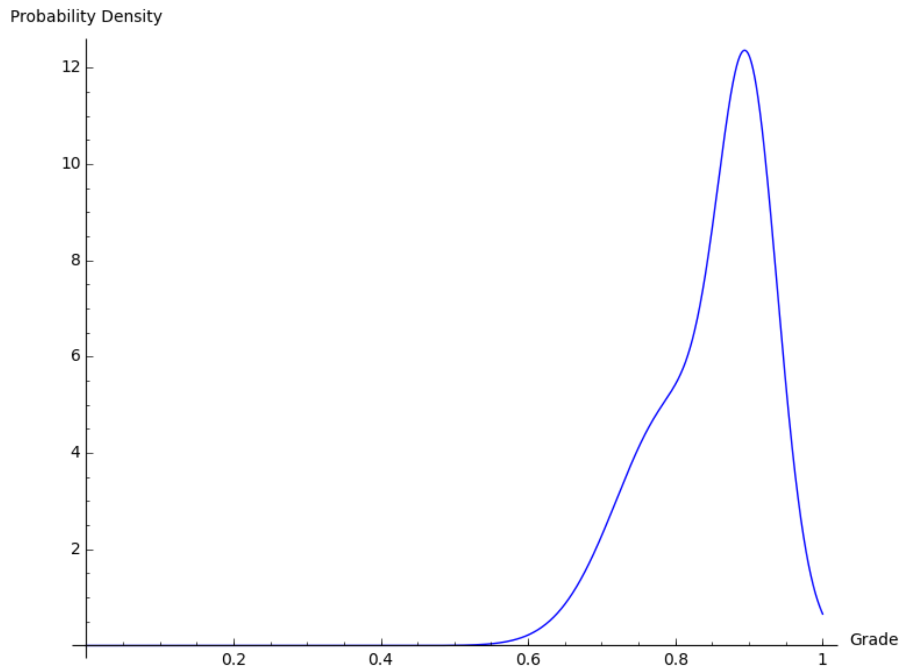
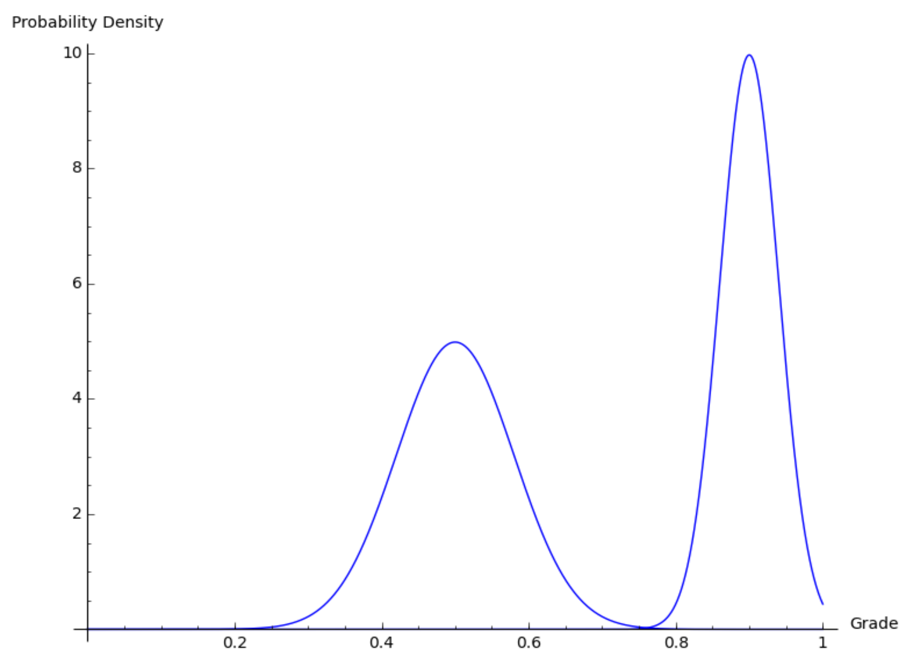
```
def GeneralPeerRank(A, alpha, beta):
    m = A.nrows()
    Xlist = [0] * m
    for i in range(0, m):
        sum = 0.0
        for j in range(0, m):
            sum += A[i,j]
        X_i = sum / m
        Xlist[i] = X_i
        X = vector(Xlist)
    fixedpoint = False
    while not fixedpoint:
        oldX = X
        X = (1-alpha-beta)*X + \
            (alpha/X.norm(1))*(A*X)
        for i in range(0, m):
            X[i] += beta - \
                (beta/m)*((A.column(i)- \
                oldX).norm(1))
        difference = X - oldX
        if abs(difference) < 10**-10:
            fixedpoint = True
    return X
```

Simulating Data

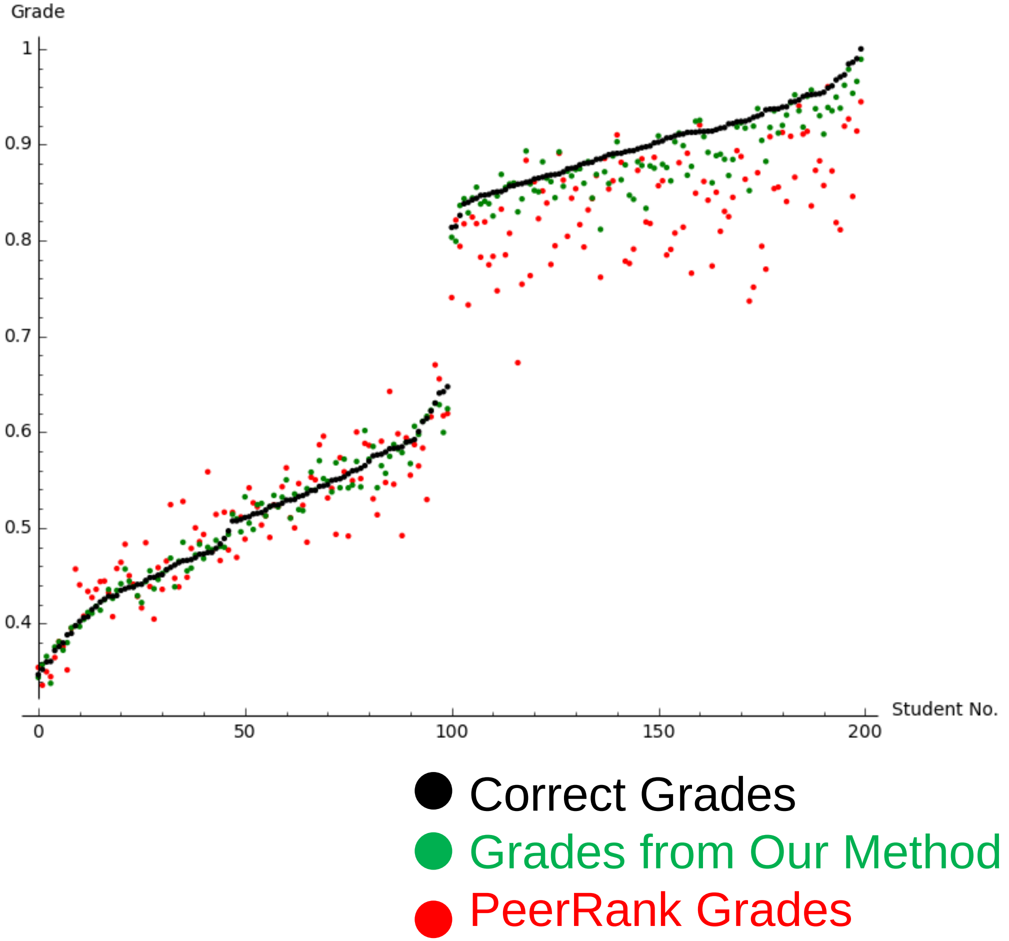
- Real grade data is not easily accessible
- Data was simulated using statistical models
  - Ground truth grades drawn from bimodal distribution
  - Accuracies drawn from normal distributions centered at grader’s grade
  - Peer grades drawn from uniform distributions using ground truth grade and accuracies





Experiments

- Experiments consisted of generating class/grade data and comparing the performance of PeerRank and our modified version against the ground truth grades.
- Variables:
  - Class size
  - Grade distribution means, standard deviations
  - Percentage of students in each group
  - Accuracy distribution standard deviation



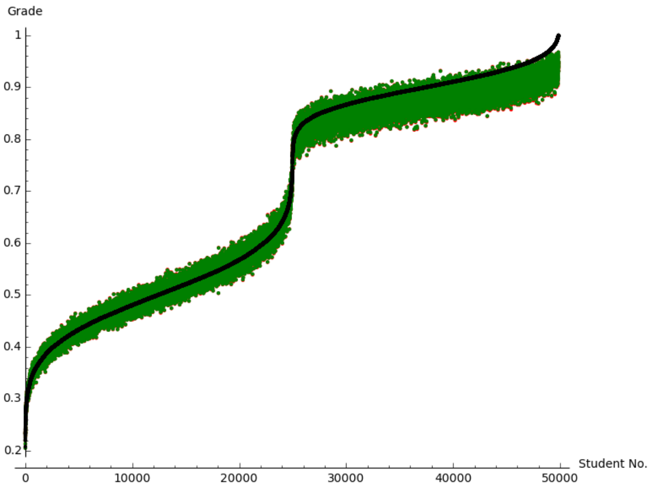
Reducing Connection Between Grade and Accuracy

- Recall: The original version of PeerRank assumes that the grader’s grade is equal to their grading accuracy.
  - Unrealistic assumption?

- Our method does assume any connection between grade and accuracy.

- How do the two versions compare as we reduce the connection between grade and accuracy?
  - We can model this reduction by increasing the standard deviation around the graders’ grades when drawing their accuracies.

Reducing Connection Between Grade and Accuracy

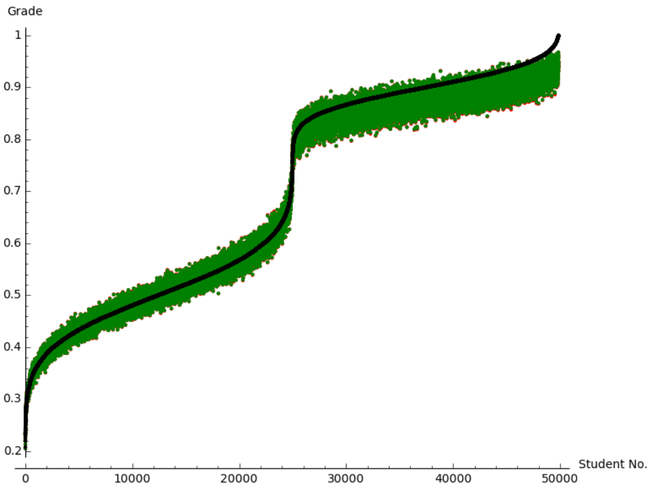


Standard Deviation = 0.02

Avg. Error Reduction < 0.1%

Correct Grades
Grades from Our Method
PeerRank Grades

Reducing Connection Between Grade and Accuracy



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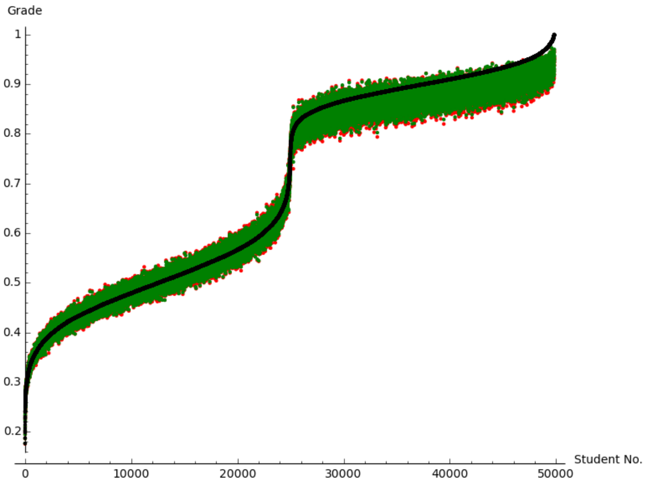
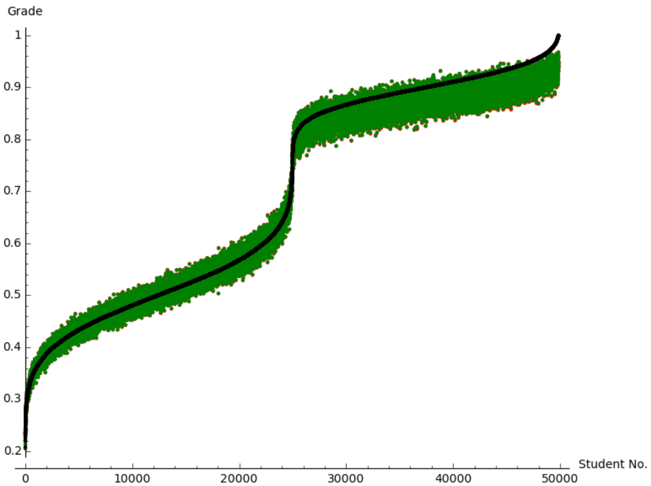


Standard Deviation = 0.10

Avg. Error Reduction ≈ 0.2%

● Correct Grades
● Grades from Our Method
● PeerRank Grades

Reducing Connection Between Grade and Accuracy



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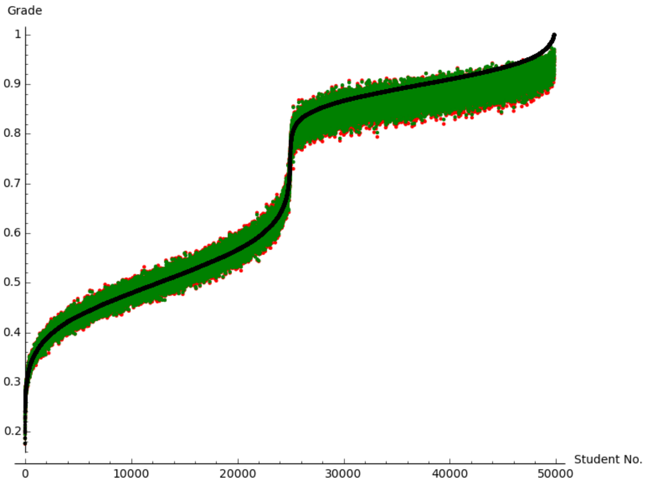
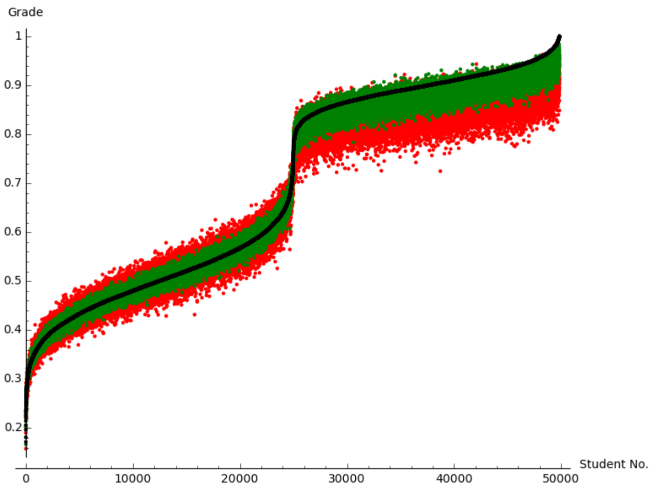


Standard Deviation = 0.10

Avg. Error Reduction ≈ 0.2%

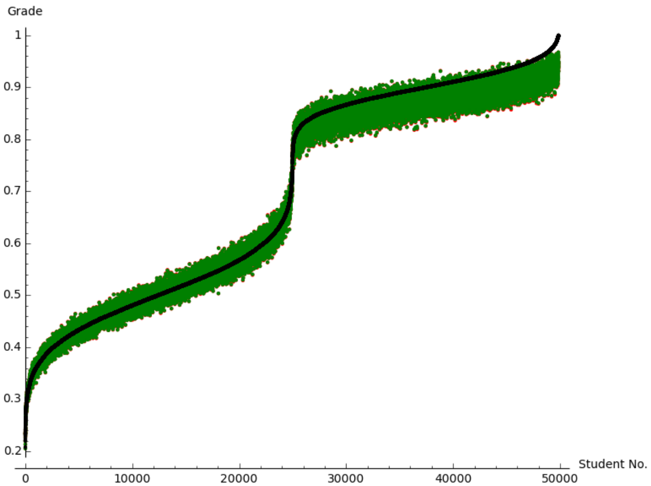


Standard Deviation = 0.50

Avg. Error Reduction ≈ 2.3%

- Correct Grades
- Grades from Our Method
- PeerRank Grades

Reducing Connection Between Grade and Accuracy



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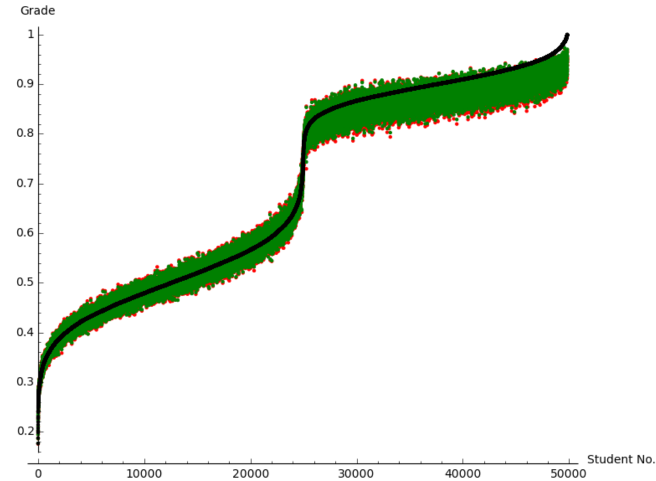

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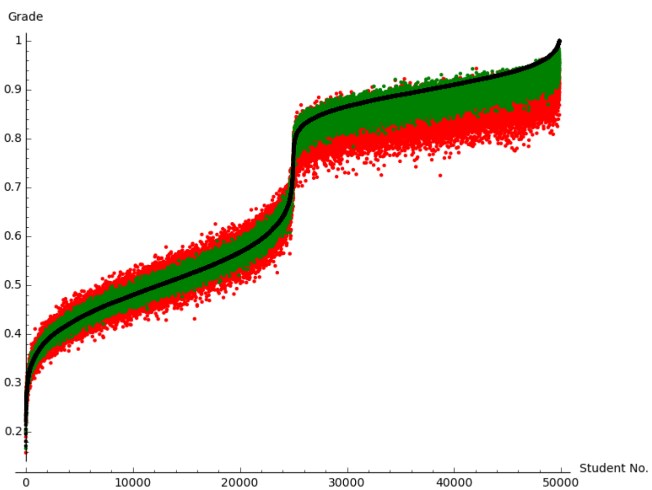

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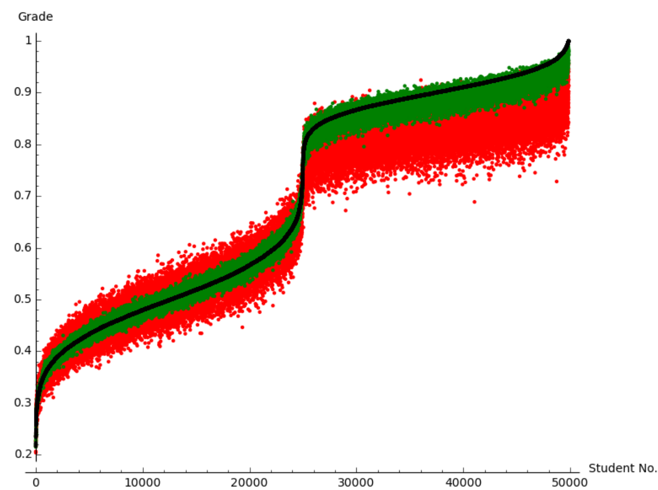


Standard Deviation = 1.0

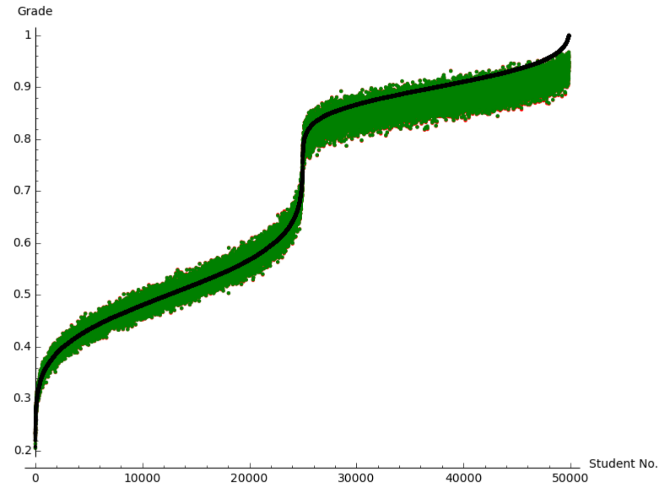
Avg. Error Reduction ≈ 4.0%

Correct Grades
Grades from Our Method
PeerRank Grades

Conclusions

- When grading accuracy is strongly correlated with the grader's grade (Walsh's assumption), our method produces grades extremely close to PeerRank.

- When grading accuracy is unrelated to the grader's grade, our method produces **more accurate** grades than PeerRank.



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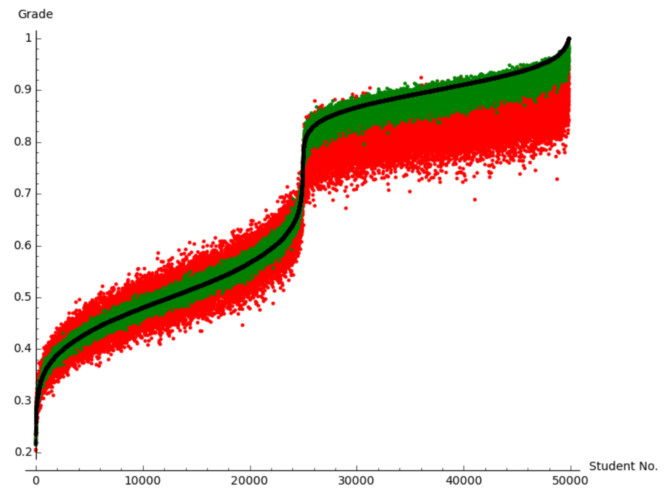

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Avg. Error Reduction ≈ 4.0%

- Correct Grades
- Grades from Our Method
- PeerRank Grades

Future Work

- Implementation of a “partial grading” scheme
  - Ignore missing grades?
  - Fill in missing grades based on known grades?
  - Best way of dividing the class?

- Additional methods of integrating ground truth
  - Instructor grades a certain number of students with a high accuracy score

Questions?